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Final, MTH 205, Fall 2015

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QUESTION 1. (i) find $\ell\{sin(x)cos(x)\}$

(ii) Find $\ell\{cos^2(x)\}$

(iii) Find $\ell\{(x+e^x)^2\}$

(iv) Find $\ell\{U(x-4)sin(x-4)\}$

(v) Find $\ell\{\int_0^x sin(3r)e^{3r}\,dr\}$

(vi) Find $\ell^{-1}\{\frac{s+4}{s^2+4s+5}\}$

(vii) Find $\ell^{-1}\{\frac{s+4}{s^2-5s-6}\}$

(viii) Use convolution ONLY to find $\ell^{-1}\{\frac{6}{s^4(s-1)}\}$

(ix) Find $\ell^{-1}\{\frac{s+9}{(s+3)^4}\}$

(x) Solve for $y(x)$: $y' + 2y = 1 - \int_0^x y(r)\,dr, y(0) = 0$

(xi) Given $y^{(2)} + 4y = 0$ has infinitely many solutions when $y(\pi/4) = 2$ and $y'(\pi/2) = a$. Find all values of a.

(xii) Solve for $y(t), x(t)$: $y^{(2)}(t) - 4x(t) = e^t - 4$ and $y'(t) + 6x'(t) = e^t$, where $y(0) = y'(0) = 1$ and $x(0) = 1$.

(xiii) Find the largest interval around $x = 0$ so that the diff. equation $\sqrt{x+6}y^{(2)} + \frac{3}{x-10}y' + 6y = \frac{1}{x-5}$ has unique solution when $y(0) = y'(0) = \pi$.

(xiv) Find the general solution to $y' = \frac{1}{2yx+x^2e^{(y^2+y)}}$

(xv) Find the general solution to $y' = \frac{(y+1)e^{(y-x)}}{x^2+2x}$

(xvi) Find the general solution to $y' = (x+y)^2 + 5(x+y+1)$

(xvii) Find the general solution to $(sin(x) + 2y - xe^y)dy + (cos(x)y - e^y + x^2 + 4e^x)dx = 0$

(xviii) Find all critical points of $y' = y^3 + 3y^2 + 2y$ and classify each as stable, semi-stable, or unstable.

(xix) Find the general solution to $y' + \frac{sin(x)+cos(x)}{sin(x)-cos(x)}y = e^x$.

(xx) An objects weights 156.8 Newton is attached to a spring having a spring constant 16N/m. At t = 0, the object is released from a point 1.5 meter above the equilibrium position with a downward velocity 1m/s and with constant external force F(t) = 56.

a) Find the equation of the motion, $x(t)$.

b) Find the phase angle ϕ and rewrite $x(t)$ using the angle ϕ.

c) Will the object ever cross the equilibrium point? Explain your answer.

d) If your answer is a NO to (c), then what should the maximum constant external force be so that the object will pass through the equilibrium point?

(xxi) Find the general solution to $y(x)$: $xy^{(2)} + y' + \frac{4}{x}y = \frac{1}{x}sec(2ln(x))$.

(xxii) A tank has a capacity of 260 liters, it contains 100 liters in which 20 grams of salt is dissolved. A mixture containing 4 grams of salt per liter is pumped into the tank at rate of 7 liter/min; the well-mixed solution is pumped out at rate 4 liter/min. Find the amount of salt, A(t), in the tank at any time t. When does an overflow occur? How much salt will be in the tank at the time it overflows? what is the concentration of the salt in the tank at the time it overflows?

(xxiii) (a) Find the general solution to $y^{(5)} + 2y^{(4)} + y^{(3)} = 0$.

(b) For the diff. equation $y^{(5)} + 2y^{(4)} + y^{(3)} = 20 + (x^2 + x^3)e^{-x}$ write down the form of y_p but do not find it.

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